

# Lecture Notes VII – Principal Component Analysis

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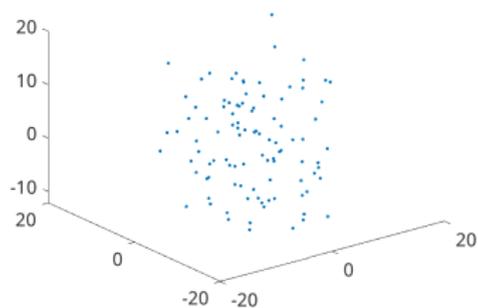
March 19, 2026



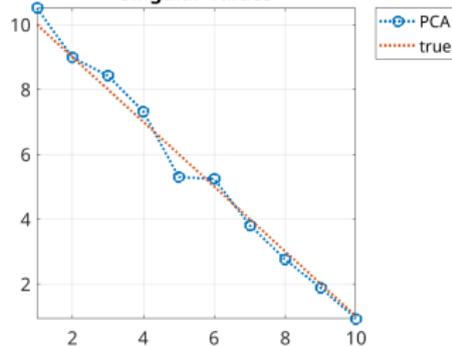
# Eigendecompositions of Variance

# Example – Gaussian data

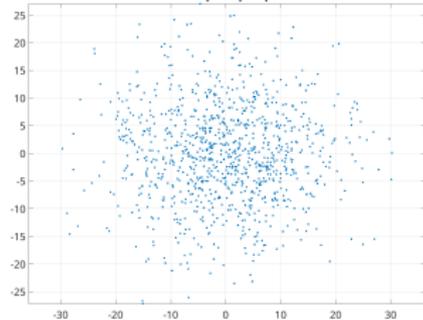
data in dimensions 1:3



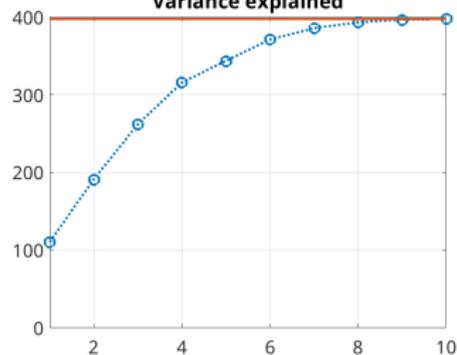
singular values



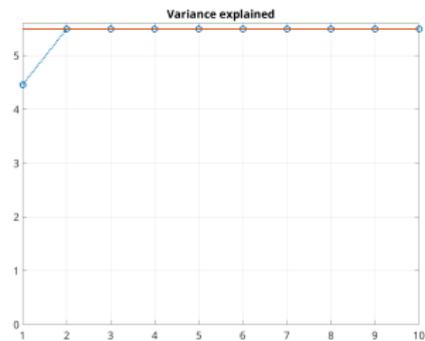
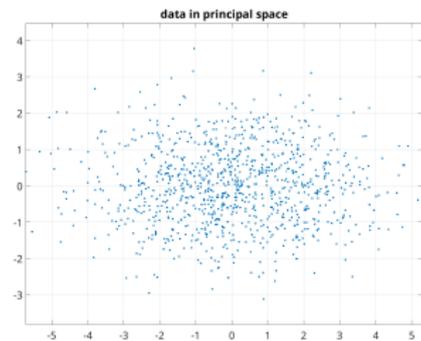
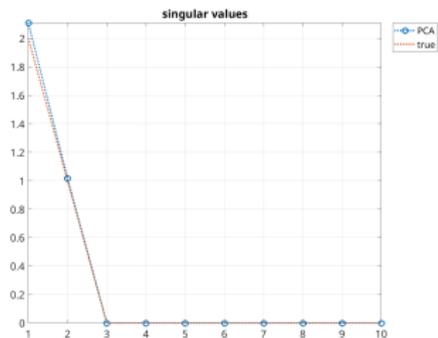
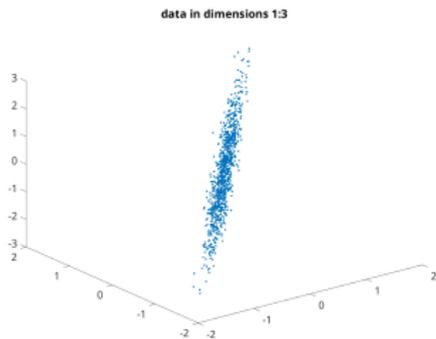
data in principal space



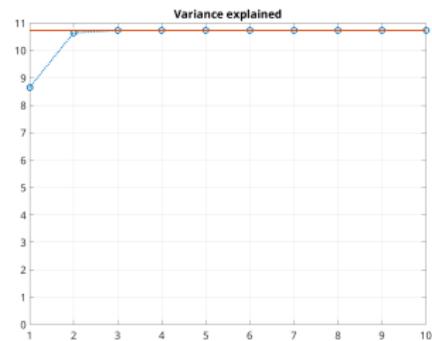
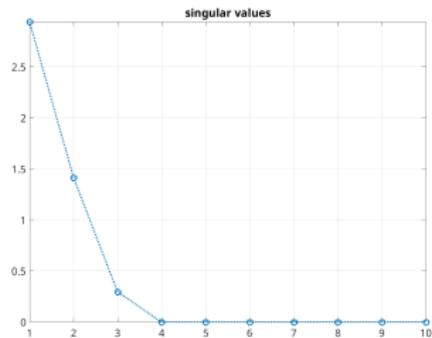
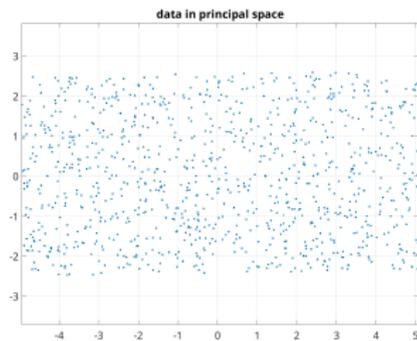
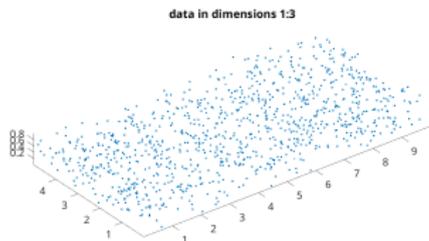
Variance explained



# Example – Gaussian data 2D

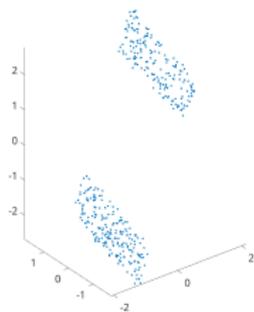


# Example – Brick

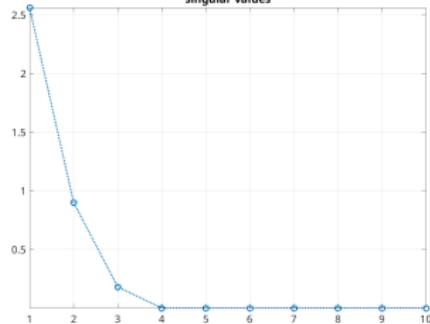


# Example – clusters

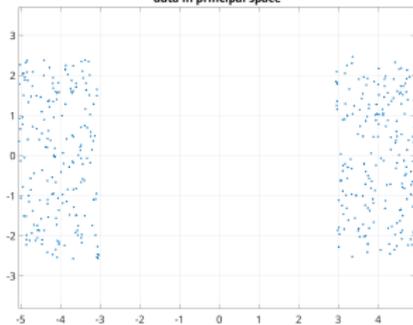
data in dimensions 1:3



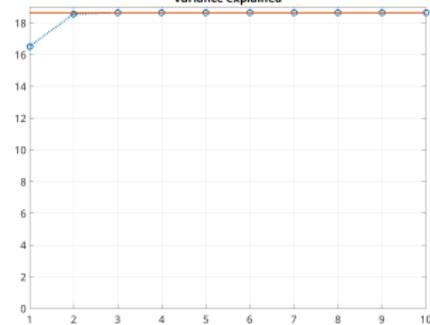
singular values



data in principal space



Variance explained



## PCA Summary

- ▶ Reduces data dimension from  $D$  to  $d$
- ▶ Linear operation (projection)
- ▶ “Optimal” linear method to reduce dimension
- ▶ Can discover if data is low-dimensional
- ▶ For clustering – recommended pre-processing: PCA in  $K - 1$  dimensions
- ▶ Limitation: fails to discover non-linear low dimensional structure